# University of Tehran Dept. of Civil Engineering <br> "Dynamics of Structures- Sample Problems" 

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1. Determine the equation of motion for the systems shown in Fig. P1 (3 cases). Homework
2. Consider a dynamic system as shown in Fig. P2. Assume that $W=10$ ton, $k=1.5 \mathrm{ton} / \mathrm{cm}$ and $c=1.0$ ton. $\mathrm{s} / \mathrm{cm}$. If the support A has a harmonic displacement $u_{\mathrm{gA}}=2.0 \cos \pi t[\mathrm{~cm}]$, compute the frequencies and eigen vectors of the system. Final Exam
3. Evaluate the frequencies and mode shapes of the structure shown in Fig. P3. Homework
4. Consider the dynamic system of Fig. P4 with damping ratio of $\xi=40 \%$. If node (1) is subjected to a harmonic load $P(t)=P_{0} \sin \omega t$ and spring (123) acts properly in both tension and compression, determine the displacement response of node (1) as $\frac{u(t)}{P_{0} / k}$ assuming $u(0)=0$, $\dot{u}(0)=0$ and $\omega=.5 \omega_{\mathrm{n}}$.
5. Evaluate the natural frequency of the beam shown in Fig. P5 using a suitable shape function. Homework
6. A 4-ton vehicle with damping of $40 \%$ and stiffness of .5 ton/cm moves across a current retard with constant speed $V$. Consider the profile of the current retard as shown in Fig. P6. Compute the maximum vertical displacement and resulted force in the vehicle's springs if: (a) $V=10 \mathrm{~km} / \mathrm{h}$ (b) $V=110 \mathrm{~km} / \mathrm{h}$ Midterm Exam- December 2002
7. A concrete bridge beam with span of 30 m and $80 \times 120 \mathrm{~cm}$ section is modelled as a simply supported beam. If a 2 -ton vehicle move across the bridge with a constant speed of $72 \mathrm{~km} / \mathrm{h}$, determine the maximum induced moment in the beam. Assume that the damping ratio is $5 \%$, Young modulus of concrete is $2.1 \mathrm{e} 2 \mathrm{ton} / \mathrm{cm}^{2}$ and concrete density is $2.5 \mathrm{ton} / \mathrm{m}^{3}$ and consider the shape function as $\varphi(x)=\sin \pi x / l$. Midterm Exam- December 2002
8. A SDOF system with mass $m$, stiffness $k$ and damping ratio $\xi=40 \%$ is subjected to an impact loading described as:

$$
P(t)= \begin{cases}P_{0} \eta(t) & ; 0 \leq t<t_{\mathrm{d}} \\ 0 & ; t \geq t_{\mathrm{d}}\end{cases}
$$

where $\eta(t)$ is shown in the diagram of Fig. P8.
(a) Show that if $\frac{t_{\mathrm{d}}}{T_{\mathrm{n}}} \leq .25$, the maximum response ratio $R_{\mathrm{d}}$ can be obtained from:

$$
R_{\mathrm{d}}=\frac{4 \pi}{3}\left(\frac{t_{\mathrm{d}}}{T_{\mathrm{n}}}\right) \exp \left(-\frac{\xi}{\sqrt{1-\xi^{2}}} \cdot \sin ^{-1} \sqrt{1-\xi^{2}}\right)
$$

(b) In the case $\frac{t_{\mathrm{d}}}{T_{\mathrm{n}}}=.25$, evaluate the value of $R_{\mathrm{d}}$ from i. The above formula, ii. Newmark method ( $\beta=1 / 6, \gamma=1 / 2$ ). To do this, evaluate the response of the system in a time domain of $\left[0, .5 T_{\mathrm{n}}\right]$ with time step $\Delta t=.1 T_{\mathrm{n}}$ using the Table P8.


Table P8

| time step <br> (i) | $\frac{u_{i}}{\left(u_{\mathrm{st}}\right)_{0}}$ | $\frac{\dot{u}_{i}}{\left(u_{\mathrm{st}}\right)_{0} \omega_{\mathrm{n}}}$ | $\frac{\ddot{u}_{i}}{\left(u_{\mathrm{st}}\right)_{0} \omega_{\mathrm{n}}^{2}}$ | $\eta_{i}$ | $\Delta \eta_{i}$ | $\frac{\Delta u_{i}}{\left(u_{\mathrm{st}}\right)_{0}}$ | $\frac{\Delta \dot{u}_{i}}{\left(u_{\mathrm{st}}\right)_{0} \omega_{\mathrm{n}}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.0 | 0.0 |  | 1.0 |  |  |  |
| 1 |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |
| 5 |  |  |  |  | -------- | -------- | -------- |

